Centre Finder

Key Stage: 4

Strand: Measures, Shape and Space

Learning Units: Basic properties of circles

Objective: To apply mathematical knowledge to solve real-life problems

Pre–requisite Knowledge:
(i) understand that the perpendicular bisector of a chord passes through the centre
(ii) understand that if the angle at circumference is a right angle, then the chord that subtends the angle is a diameter

Resources Requires: 3D printers, 3D design software (e.g. Tinkercad, SketchUp)

Relationship with other KLA(s) in STEM Education:
The centre finder is a tool that could be related to the content of the learning elements "(K5) Tools and Equipment" and "(K6) Production Process" in the knowledge context "Operations and Manufacturing" of Technology Education Key Learning Area at Secondary 1 and Secondary 3. It could also be related to the learning content of the topic "Design in practice" of the elective subject Design and Applied Technology in senior secondary.

Background information:
In real-life situation, people need to mark the centre of a circular wooden block to do construction work, or mark the centre of a ceiling to install a suspension lamp, or locate the centre of a large circular water tunnel to install a monitoring equipment to measure the speed of the water flow. Centre finder is a useful tool to help people locate the centre of a circle. Students could understand why the centre finder works after they have learned the basic properties of circles in the senior secondary Mathematics curriculum. Apart from that, students have to make use of a 3D printer and a 3D design software to design and make their own centre finders.

Description of the activity:

Activity 1: How and Why does a centre finder work?

1. The teacher may explain to students the need for the use of a tool to locate the centre of circle as mentioned in the background information above.
2. The teacher may introduce the task of designing a suitable tool, i.e. the centre finder, to locate the centre of a circular can in the next activity. Before the task, students may watch the following video to understand how to produce a centre finder and how to use it. They may then discuss why the tool works with their classmates. Students are expected to use the same principle to design and make their own centre finders in the next activity.
   https://www.youtube.com/watch?v=j3xmQ8rYk8c

Notes for teachers:

1. The teacher may guide students to guess the geometric properties of the edge between the arms. By figuring out the edge is in fact the angle bisector of the angle formed by the arms, the teacher may discuss with students the geometric relation between the centre of a circle (the circular object) and the angle bisector (the edge) by considering the theorem "the line joining the centre and an external point of a circle bisects the angle included by the two tangents from the point to the circle", where the edges of the arms and the vertex of the angle formed by the edges are respectively the tangents and the external point concerned. Therefore, the above theorem shows that the line joining the centre and the vertex coincides with the edge between the arms (the angle bisector), and hence the centre lies on the edge. It also means that the line segment formed by the edge in the circle is a diameter. By drawing two different diameters of the circle, the centre is located.

   The teacher may also introduce the angle bisector theorem which states that all the points lying on the angle bisector are equidistant from the arms as a geometric property of the angle bisector to further enrich students' understanding of the angle bisector.

2. The teacher may discuss with students the restrictions of producing the same tool in a normal classroom setting. For example, in the video, the production of the centre finder requires suitable instruments and materials that may not be available. The teacher may suggest students using a 3D design software (e.g. Tinkercad, SketchUp) to design their own center finders and make use of a 3D printer to produce the products in the next activity.
3. It should be aware that angle between two arms of the Circle Centre Finder Tool in the YouTube video, that is 60°, is not critical. The teacher may ask students to think about this point and suggest other angle for replacement. Of course, it is not practical for the angle to be 10° or 160°, etc.

Activity 2: Design and make a centre finder

1. The centre finder may consist of different parts, namely a V-shaped prism and a rectangular prism.
2. The teacher demonstrates how to make a V-shaped prism from the text "V" by using a 3D design software (Figure 1). (The dimension of the V-shaped prism in Figure 1 is 180mm by 150mm by 3mm, for example, and the font of the text "V" is Sans.)

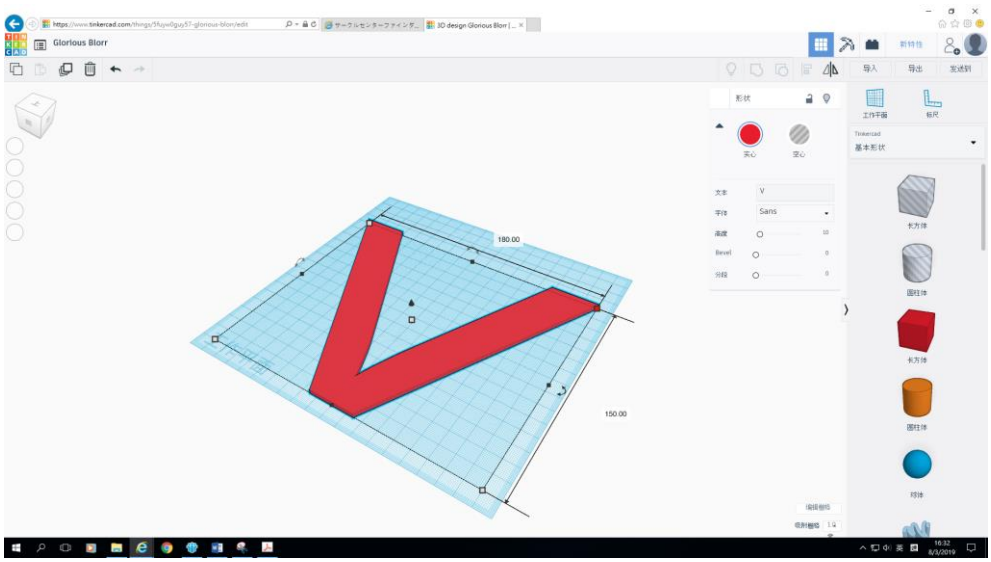


Figure 1

3. The teacher demonstrates how to make a rectangular prism and how to place it on the top of the V-shaped prism (Figure 2). (The dimension of the rectangular prism in Figure 2 is 150mm by 15mm by 3mm, for example. The rectangular prism is placed on the top of the V-shaped prism with one side on the top of the angle bisector of the arms. The V-shaped prism and the rectangular prism are then merged into one object.)

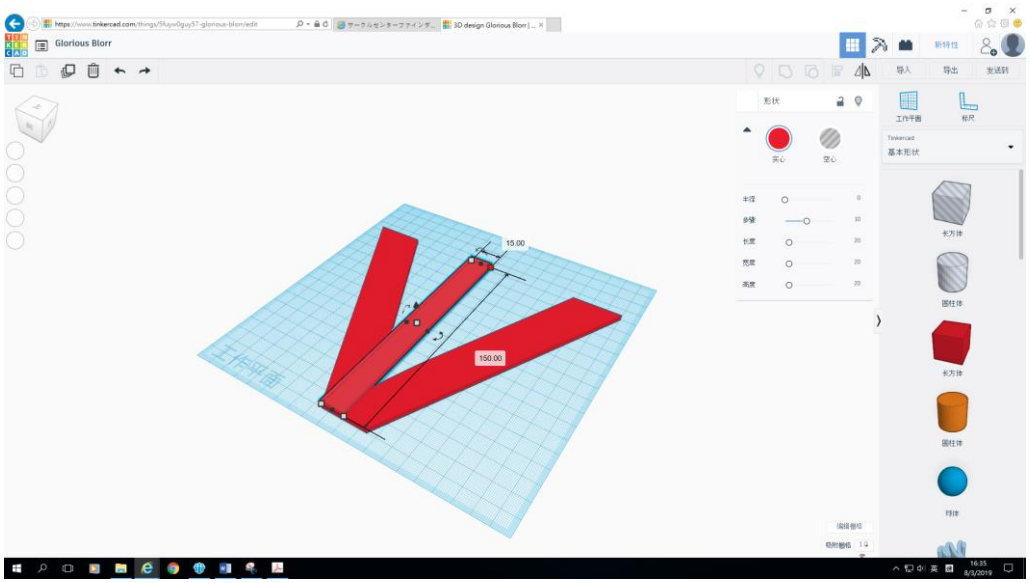


Figure 2

4. The teacher may ask students if there is any problem to print the aforesaid object through a 3D printer. As students may not be able to aware that some part of the rectangular prism cannot be printed out as it lies above the ground. Students have to modify the object for the 3D printer to produce a centre finder. For example, students may place the rectangular prism on the ground and get thicker for the V-shaped prism with dimension 180mm by 150mm by 6mm (Figure 3).

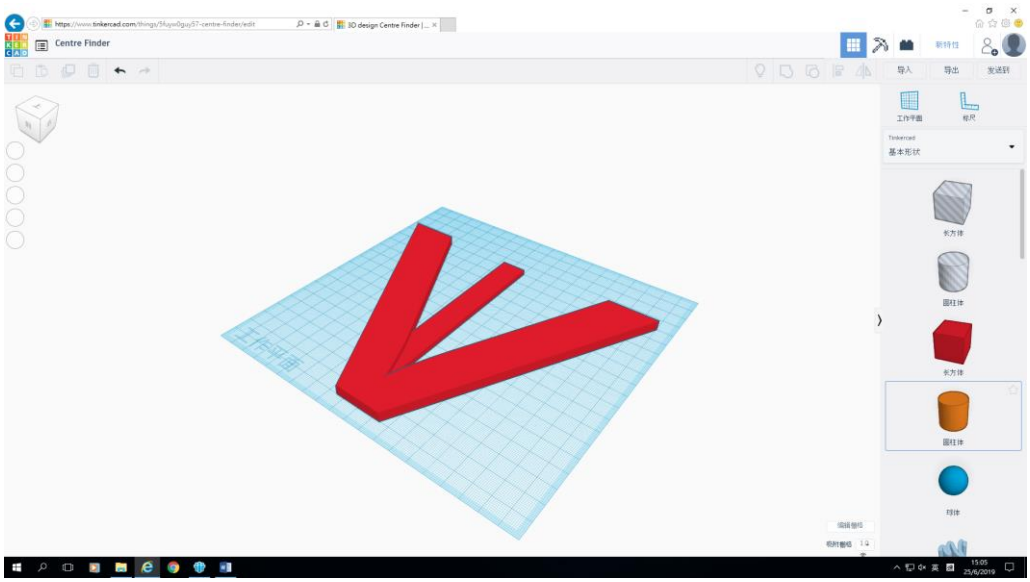


Figure 3

5. The teacher collects students' designs and exports them to the 3D printer. Students use their own centre finders to locate the centre of a circular can.

Notes for teachers:

1. The design of suitable centre finder by using a 3D design software may be referred as an activity in the Topic “Design in practice” in the curriculum of the elective subject Design and Applied Technology in senior secondary.
2. The teacher may remind students that the 3D design software may use millimetre as the unit. They may need to convert the length to centimetres.
3. Students may adjust the dimension of their own designs but they should be aware of the limitation of the print dimensions of the 3D printer.
4. Apart from using the text “V”, students may use other methods to produce a V-shaped prism. For example, they may draw a rectangular prism and cut out two right-angled prisms at two corners, and also cut a prism with base as an isosceles triangle to form a V-shaped prism (Figure 4).

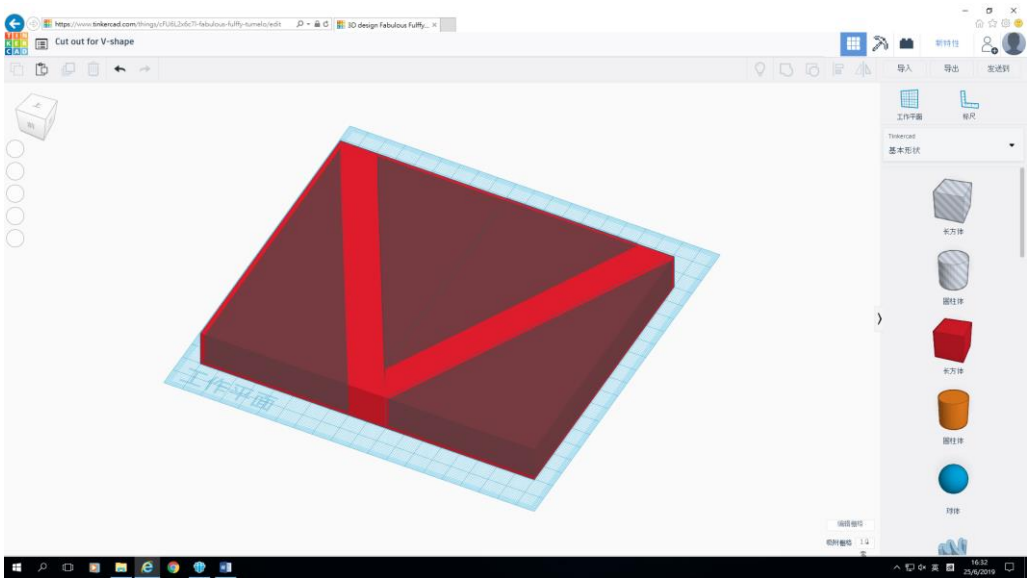


Figure 4

5. Students may watch the following video which adopt the same principle with different design. Students may produce the same tool with a 3D design software and a 3-D printer.
   https://www.youtube.com/watch?v=BZQOpBMJTLw

6. The following theorems that students should have learnt are related to locating the centre of a circle.
   - Theorem 1: The perpendicular bisector of a chord passes through the centre.
   - Theorem 2: If the angle at the circumference is a right angle, then the chord that substends the angle is a diameter.

   Students may use Theorem 1 or Theorem 2 to design their own centre finders.
7. By using Theorem 1, students may deign the T-shaped tool to locate the centre of a circle (Figure 5).

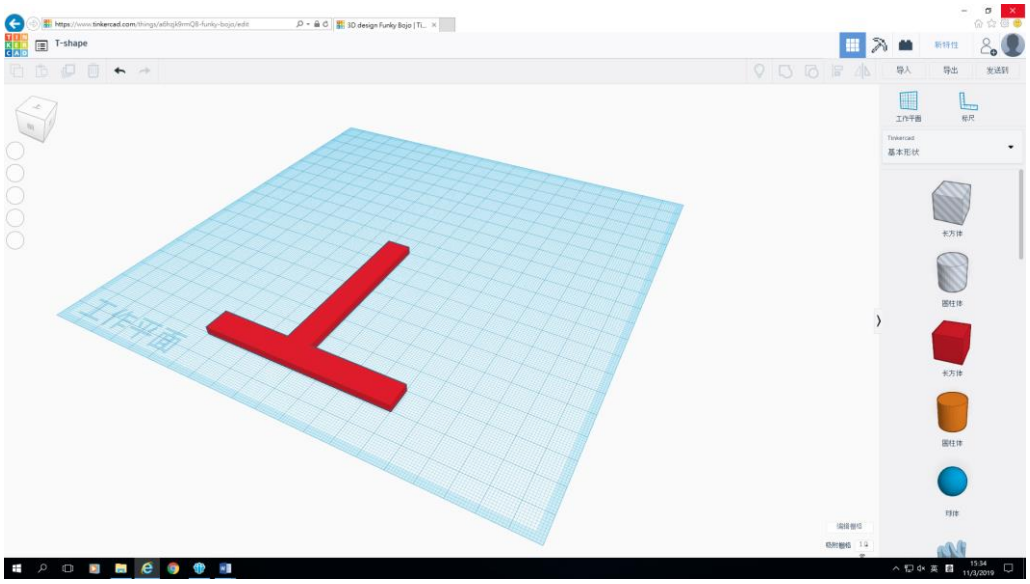


Figure 5

In using the T-shaped, students have to mark two diameters to locate the centre of a circle (Figure 6)

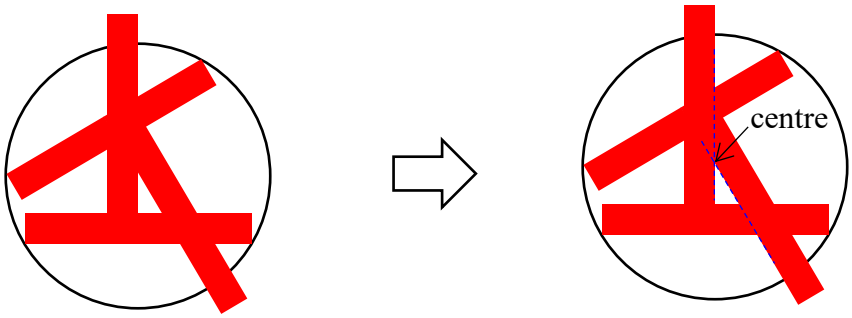


Figure 6

8. By using Theorem 2, students may deign the L-shaped tool to locate the centre of a circle (Figure 7).

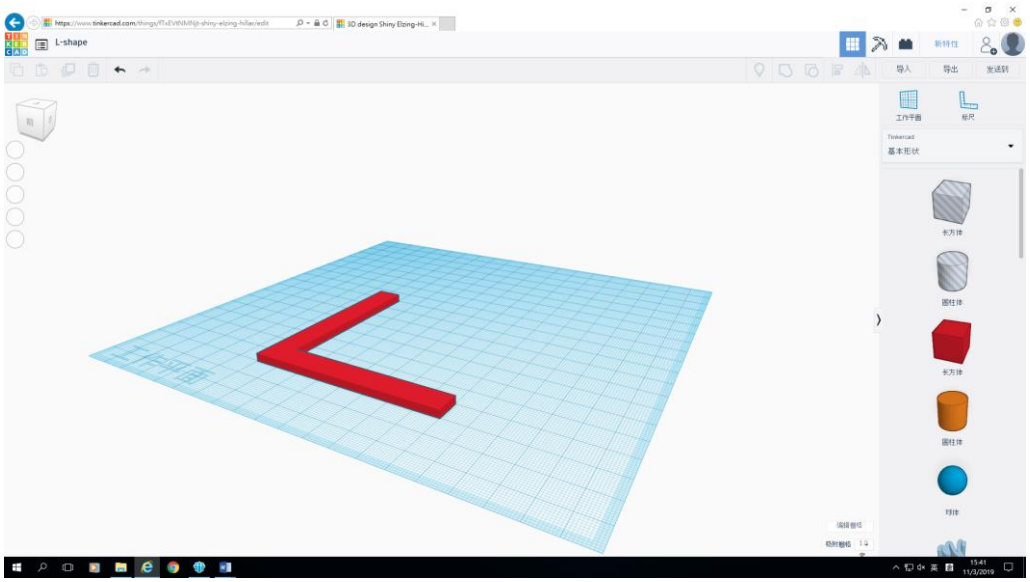


Figure 7

In using the L-shaped, students have to mark pairs of end points to draw two diameters to locate the centre of a circle (Figure 8)

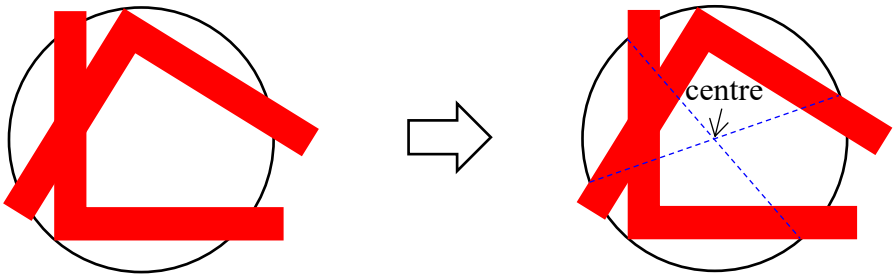
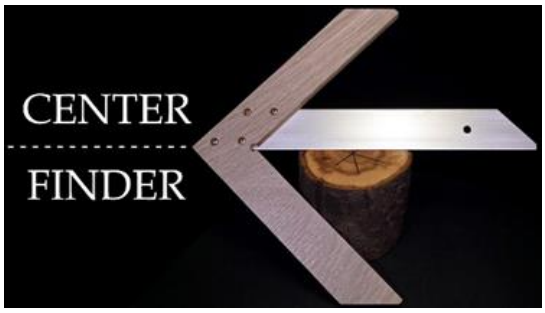


Figure 8

9. The teacher may use the following video as an extension activity for students to learn that if the arms of the V-shaped tool be set as a right angle (Time 4:30 – 5:15). It should be noted that it is an alternative way to mark the centre of a square by finding the intersection point of the diagonals.
https://www.youtube.com/watch?v=4OttW4f1SsM



Further questions may be asked after students have watched the video. For example, Figure 9 shows the way to locate the centre of a square *ABCD* by tilting the square to get a line *QJ* passing through the centre *I*. Students have to explain why it works. (*Hint: Prove that* $\angle AQI = 45°$)

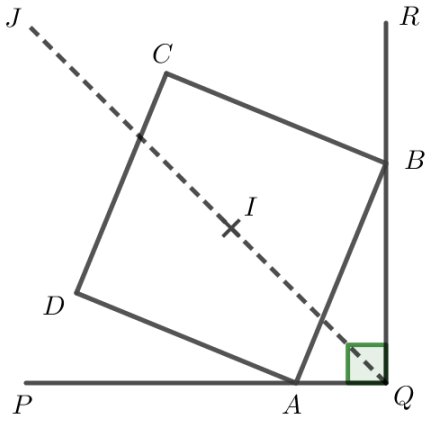


Figure 9

Proof: $\angle AQB + \angle AIB = 90° + 90°$

$= 180°$

A, *Q*, *B* and *I* are concyclic. (opp.∠s supp.)

$\angle ABI = 45°$ (prop. of square)

$\angle AQI = 45°$ (∠s in the same segment)

∴ *QJ* is the line passing through the centre of the square *ABCD*.